

# 1 Conditional Probability

## 1.1 Concepts

1. The probability of an event  $A$  given that we know an event  $B$  occurred is denoted as  $P(A|B)$  and the formula is

$$P(A|B) := \frac{P(A \cap B)}{P(B)}.$$

Sometimes, we do not know  $P(B)$  and we have to rewrite it as

$$P(B) = P(B \cap A) + P(B \cap \bar{A}) = P(B|A)P(A) + P(B|\bar{A})P(\bar{A})$$

## 1.2 Examples

2. I flip a fair coin 12 times. What is the probability that exactly 10 heads appear given that at least two heads appeared?

## 1.3 Problems

3. True     False     If  $A \subset B$ , then  $P(A|B) = P(A)/P(B)$ .
4. True     False     If  $P(A|B) = P(B|A)$ , then  $P(A) = P(B)$ .
5. Suppose that 80% of students have taken Calculus and of those, only 20% become math graduate students. Suppose that 30% of all students become math graduate students. Let  $C$  be the set of students who have taken Calculus and let  $G$  be the set of students who become math graduate students. Represent 80%, 20%, 30% in terms of probabilities (e.g.  $P(A)$  or  $P(A|B)$ ).
6. Using the previous problem, what is the probability that someone has not taken calculus and is a math graduate student? What is the probability that a math graduate student has not taken calculus?
7. Out of 330 male students and 270 female students in 10B, 210 of the men and 180 of the women took 10A with Zvezda last semester. What is the probability that a randomly person is a female given that they took 10A with Zvezda?

8. There are two boxes. One has 10 red balls and no white balls and one has 5 red and 5 white balls. I randomly pick a box then randomly pick out a ball. What is the probability I pick out a red ball?
9. What is the probability that I have a full house (one triple, one pair) in my 5 card hand if I know that I have a triple (but not 4 of a kind)?

## 2 Review

10. How many ways can you rearrange the letters in BERKELEY?
11. There are 72 students trying to get into 3 of my sections. There are 27, 20, 25 openings respectively. How many ways are there for these students to enroll?
12. How many ways can I put 20 Tootsie rolls into 5 goodie bags so that each goodie bag has at least 2 Tootsie roll?
13. Show that when you place 9 coins on an  $8 \times 10$  boards, at least two coins must be on the same row.
14. How many different three-letter initials contain *A*?
15. How many license plates with 3 digits followed by 3 letters do not contain the both the number 0 and the letter *O* (it could have an *O* or a 0 but not both).
16. Prove that  $\binom{n-1}{r-1} + \binom{n-1}{r} = \binom{n}{r}$  in two different ways.
17. Prove that  $\sum_{k=0}^n 5^k \binom{n}{k} = 5^0 \binom{n}{0} + 5^1 \binom{n}{1} + \cdots + 5^n \binom{n}{n} = 6^n$ .
18. How many ways can I split up 30 distinguishable students into 6 groups each of size 5?
19. Find a formula for  $1 + 2 + 4 + \cdots + 2^n$  and prove it.